Ex 1.2

Q.1. Find the real values of x and y in each of the following.

i) $x+iy+2-3i = i(5-i)(3+4i)$

Sol:- $(x+2)+(y-3)i = i(5(3+4i)-i(3+4i))$

$(x+2)+(y-3)i = i(15+20i-3i-4i^2)$

$(x+2)+(y-3)i = i(15+17i+4)$

$(x+2)+(y-3)i = i(19+17i)$

$(x+2)+(y-3)i = 19i+17i^2$

$(x+2)+(y-3)i = 19i-17$

Equating real and img parts

$x+2 = -17 \rightarrow (1)$

$y-3 = 19 \rightarrow (2)$

from (1)

$x+2 = -17$

$x = -17-2$

$\boxed{x = -19}$

from (2)

$y-3 = 19$

$y = 19+3$

$\boxed{y = 22}$

(ii) $(x+iy)(1-i) = (2-3i)(-5+5i)(-i\frac{3}{5})$

Sol:-

$(x+iy)(1-i) = (2-3i)(-5+5i)(-i\frac{3}{5})$

$x(1-i)+iy(1-i) = (2(-5+5i)-3i(-5+5i))(-\frac{3i}{5})$

$x-xi+yi-i^2y = (-10+10i+15i-15i^2)(-\frac{3i}{5})$

$x-xi+yi+y = (-10+25i+15)(-\frac{3i}{5})$

$(x+y)+(-x+y)i = (5+25i)(-\frac{3}{5}i)$

$(x+y)+(-x+y)i = -(5)\frac{3i}{5}+25i(-\frac{3i}{5})$

$(x+y)+(-x+y)i = -3i+5i(-3i)$

$(x+y)+(-x+y)i = -3i-15i^2$

$(x+y)+(-x+y)i = -3i+15$

$(x+y)+(-x+y)i = 15-3i$

Equating real and img parts

$x+y = 15 \longrightarrow (1)$

$-x+y = -3 \longrightarrow (2)$

Adding both equations

$$\begin{aligned} x+y &= 15 \\ -x+y &= -3 \\ \hline 2y &= 12 \end{aligned}$$

$y = \frac{12}{2}$

$\boxed{y=6}$

put $y=6$ in equ (1)

$$6+y=15$$

$$y=15-6$$

$$\boxed{y=9}$$

(iii) $\dfrac{x}{2+i}+\dfrac{y}{3-i}=4+5i$

Sol:-

$$\frac{x}{2+i}+\frac{y}{3-i}=4+5i$$

$$\frac{x(3-i)+y(2+i)}{(2+i)(3-i)}=4+5i$$

$$\frac{3x-xi+2y+yi}{2(3-i)+i(3-i)}=4+5i$$

$$\frac{(3x+2y)+(y-x)i}{6-2i+3i-i^2}=4+5i$$

$$\frac{(3x+2y)+(y-x)i}{6+i+1}=4+5i$$

$$\frac{(3x+2y)+(y-x)i}{7+i}=4+5i$$

$$(3x+2y)+(y-x)i=(4+5i)(7+i)$$

$$(3x+2y)+(y-x)i=4(7+i)+5i(7+i)$$

$$(3x+2y)+(y-x)i=28+4i+35i+5i^2$$

$(3x+2y)+(y-x)i = 28+39i-5$

$(3x+2y)+(y-x)i = 23+39i$

Equating real and img parts

$3x+2y = 23 \longrightarrow ①$

$y-x = 39 \longrightarrow ②$

multiply by "3" to equ ②

$3y-3x = 117 \longrightarrow ③$

Adding equ ① and ③

$3x+2y = 23$

$-3x+3y = 117$

$5y = 140$

$y = \frac{140}{5}$

$\boxed{y = 28}$

Put $y = 28$ in equ ②

$28 - y = 39$

$-y = 39-28$

$-y = 11$

$\boxed{y = -11}$

Q2:- If $z_1 = -13 + 24i$ and $z_2 = x + yi$, find real values of x and y such that $z_1 - z_2 = -27 + 15i$

Sol:-

We are given

$z_1 = -13 + 24i$

$z_2 = x + yi$

Now,

$z_1 - z_2 = -27 + 15i$

$(-13 + 24i) - (x + yi) = -27 + 15i$

$-13 + 24i - x - yi = -27 + 15i$

$(-13 - x) + (24 - y)i = -27 + 15i$

Equating real and img parts.

$-13 - x = -27 \longrightarrow (1)$

$24 - y = 15 \longrightarrow (2)$

from (1)

$-13 - x = -27$

$-x = -27 + 13$

$-x = -14$

$\boxed{x = 14}$

from (2)

$24 - y = 15$

$-y = 15 - 24$

$-y = -9$

$\boxed{y = 9}$

Q3 Find the real value of x and y if:

(i) $(x+iy)^2 = 25 + 60i$

Sol:-

$(x+iy)^2 = 25 + 60i$

$x^2 + i^2y^2 + 2xyi = 25 + 60i$

$x^2 - y^2 + 2xyi = 25 + 60i$

Equating Real and img parts.

$x^2 - y^2 = 25 \longrightarrow (1)$

$2xy = 60 \longrightarrow (2)$

As we know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= (x^2-y^2)^2 + (2xy)^2$

$= (25)^2 + (60)^2$

$= 625 + 3600$

$(x^2+y^2)^2 = 4225$

$\sqrt{(x^2+y^2)^2} = \sqrt{4225}$

$x^2 + y^2 = 65 \longrightarrow (3)$

Adding ① and ③

$$x^2 - y^2 = 25$$
$$x^2 + y^2 = 65$$
$$2x^2 = 90$$
$$x^2 = \frac{90}{2}$$
$$\sqrt{x^2} = \sqrt{45}$$
$$x = \pm\sqrt{9 \times 5}$$
$$\boxed{x = \pm 3\sqrt{5}}$$

put $x = 3\sqrt{5}$ in equ ①

$$(3\sqrt{5})^2 - y^2 = 25$$
$$9(5) - y^2 = 25$$
$$45 - y^2 = 25$$
$$-y^2 = 25 - 45$$
$$-y^2 = -20$$
$$\sqrt{y^2} = \sqrt{20}$$
$$y = \pm\sqrt{4 \times 5}$$
$$\boxed{y = \pm 2\sqrt{5}}$$

(ii) $(x + iy)^2 = 64 + 48i$

Sol:-

$$(x + iy)^2 = 64 + 48i$$
$$x^2 + i^2y^2 + 2xyi = 64 + 48i$$

$x^2 - y^2 + 2xyi = 64 + 48i$

Equating Real and img parts

$x^2 - y^2 = 64 \longrightarrow (1)$

$2xy = 48 \longrightarrow (2)$

As we know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$

$= (64)^2 + (48)^2$

$= 4096 + 2304$

$(x^2+y^2)^2 = 6400$

$\sqrt{(x^2+y^2)^2} = \sqrt{6400}$

$x^2 + y^2 = 80 \longrightarrow (3)$

Adding equ (1) and (3)

$$\begin{aligned} x^2 - \cancel{y^2} &= 64 \\ x^2 + \cancel{y^2} &= 80 \\ \hline 2x^2 &= 144 \end{aligned}$$

$x^2 = \frac{144}{2}$

$x^2 = 72$

$\sqrt{x^2} = \sqrt{72}$

$\boxed{x = \pm 6\sqrt{2}}$

put $x = 6\sqrt{2}$ in equ (1)

$(6\sqrt{2})^2 - y^2 = 64$

$36(2) - y^2 = 64$

$$72 - y^2 = 64$$

$$-y^2 = 64 - 72$$

$$\cancel{-}y^2 = \cancel{-}8$$

$$y^2 = 8$$

$$\sqrt{y^2} = \sqrt{8}$$

$$\boxed{y = \pm 2\sqrt{2}}$$

(iii) $(x + iy)^2 = \dfrac{2i - 3}{3 + i}$

Sol:-

$$x^2 + i^2y^2 + 2xyi = \frac{2i-3}{3+i} \times \frac{3-i}{3-i}$$

$$x^2 - y^2 + 2xyi = \frac{(2i-3)(3-i)}{(3+i)(3-i)}$$

$$x^2 - y^2 + 2xyi = \frac{2i(3-i) - 3(3-i)}{(3)^2 - (i)^2}$$

$$x^2 - y^2 + 2xyi = \frac{6i - 2i^2 - 9 + 3i}{9 - i^2}$$

$$x^2 - y^2 + 2xyi = \frac{9i + 2 - 9}{9 - (-1)}$$

$$(x^2 - y^2) + 2xyi = \frac{9i - 7}{9 + 1}$$

$$(x^2 - y^2) + 2xyi = \frac{9i - 7}{10}$$

$$(x^2 - y^2) + (2xyi) = \frac{9}{10}i - \frac{7}{10}$$

Equating Real and img Parts

$x^2 - y^2 = -\frac{7}{10} \longrightarrow (1)$

$2xy = \frac{9}{10} \longrightarrow (2)$

As we know that

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= \left(-\frac{7}{10}\right)^2 + \left(\frac{9}{10}\right)^2$

$= \frac{49}{100} + \frac{81}{100}$

$= \frac{49+81}{100}$

$= \frac{130}{100}$

$\sqrt{(x^2+y^2)^2} = \sqrt{\frac{13}{10}}$

$x^2 + y^2 = 1.14 \longrightarrow (3)$

Adding (1) and (3)

$x^2 - \cancel{y^2} = -0.7$

$x^2 + \cancel{y^2} = 1.14$

$2x^2 = 0.44$

$x^2 = \frac{0.44}{2}$

$x^2 = 0.22$

$$\sqrt{x^2} = \sqrt{0.22}$$

$$\boxed{x = \pm 0.46}$$

Put $x = 0.46$ in equ (1)

$$(0.46)^2 - y^2 = -0.7$$

$$0.2116 - y^2 = -0.7$$

$$-y^2 = -0.7 - 0.2116$$

$$-y^2 = -0.9116$$

$$\sqrt{y^2} = \sqrt{0.9116}$$

$$\boxed{y = \pm 0.95}$$

Q4 If $z_1 = 2+3i$ and $z_2 = 1-\alpha$, find the real value of α such that $Im(z_1 z_2) = 7$

Sol:-

$$Im(z_1 z_2) = 7$$

Given that

$$z_1 = 2+3i \quad , \quad z_2 = 1-\alpha$$

$$Im((2+3i)(1-\alpha)) = 7$$

$$Im(2(1-\alpha) + 3i(1-\alpha)) = 7$$

$$Im(2-2\alpha + 3i(1-\alpha)) = 7$$

$$3(1-\alpha) = 7$$

$$1-\alpha = \frac{7}{3}$$

$$-\alpha = \frac{7}{3} - 1$$

$$-\alpha = \frac{7-3}{3}$$

$$-\alpha = \frac{4}{3}$$

$$\boxed{\alpha = -4/3}$$

Q5 If $z_1 = x + yi$ and $z_2 = a + bi$, find x, y, a and b such that $z_1 + z_2 = 10 + 4i$ and $z_1 - z_2 = 6 + 2i$

Sol:-

We are given

$z_1 = x + yi$, $z_2 = a + bi$

$$z_1 + z_2 = 10 + 4i$$

$$x + yi + a + bi = 10 + 4i$$

$$(x+a) + (y+b)i = 10 + 4i$$

Equating Real and img numbers

$$x + a = 10 \rightarrow (1)$$

$$y + b = 4 \rightarrow (2).$$

Now, again

$$z_1 - z_2 = 6 + 2i$$

$$(x+yi) - (a+bi) = 6 + 2i$$

$$x + yi - a - bi = 6 + 2i$$

$$(x-a) + (y-b)i = 6 + 2i$$

Equating real and img number

$x - a = 6 \longrightarrow$ (3)

$y - b = 2 \longrightarrow$ (4)

Adding (1) and (3)

$x + \cancel{a} = 10$

$x - \cancel{a} = 6$

$2x = 16$

$x = \frac{16}{2}$

$\boxed{x = 8}$

Put $x = 8$ in equ (1)

$8 + a = 10$

$a = 10 - 8$

$\boxed{a = 2}$

Now again,

adding (2) and (4)

$y + \cancel{b} = 4$

$y - \cancel{b} = 2$

$2y = 6$

$y = \frac{6}{2}$

$\boxed{y = 3}$

Put $y = 3$ in equ (2)

$3 + b = 4$

$b = 4 - 3$

$\boxed{b = 1}$

Q6 Show that $\forall z_1, z_2 \in C$, $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Proof:-

Let $z_1 = a+bi$, $z_2 = c+di$

and

$\bar{z}_1 = a-bi$ $\bar{z}_2 = c-di$

$LHS = \overline{z_1 z_2}$

$z_1 z_2 = (a+bi)(c+di)$

$z_1 z_2 = a(c+di) + bi(c+di)$

$= ac + adi + bci + bdi^2$

$= ac + (ad+bc)i - bd$

$= (ac-bd) + (ad+bc)i$

$\overline{z_1 z_2} = \overline{(ac-bd) + (ad+bc)i}$

$= (ac-bd) - (ad+bc)i$

Now take

$RHS = \bar{z}_1 . \bar{z}_2$

$\bar{z}_1 \bar{z}_2 = (a-bi)(c-di)$

$= a(c-di) - bi(c-di)$

$= ac - adi - bci + bdi^2$

$= ac - (ad+bc)i - bd$ $\because i^2 = -1$

$= (ac-bd) - (ad+bc)i$

Hence $LHS = RHS$

$\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$.

Q7 Find the Square root of the following complex numbers.

(i) $7 - 24i$

Sol:-

By using formula

$$\sqrt{x+iy} = \pm\left(\sqrt{\frac{\sqrt{x^2+y^2}+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{\sqrt{x^2+y^2}-x}{2}}\right)$$

Let

$$x+iy = 7-24i$$

$\Rightarrow$ $x = 7$ and $y = -24 < 0$

$$|7-24i| = \sqrt{7^2+24^2} = \sqrt{49+576}$$

$$= \sqrt{625} = 25$$

Applying Square root formula,

$$\sqrt{7-24i} = \pm\left(\sqrt{\frac{25+7}{2}} + \left(\frac{-24i}{|24|}\right)\sqrt{\frac{25-7}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{32}{2}} - i\sqrt{\frac{18}{2}}\right)$$

$$= \pm\left(\sqrt{16} - i\sqrt{9}\right)$$

$$= \pm(4 - 3i)$$

(ii) $8-6i$

Sol:-

Let $x+iy = 8-6i$

$\Rightarrow x = 8$ and $y = -6 < 0$

$|8-6i| = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$= 10$

Applying square root formula

$$\sqrt{8-6i} = \pm\left(\sqrt{\frac{10+8}{2}} + \frac{(-6i)}{|6|}\sqrt{\frac{10-8}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{18}{2}} - i\sqrt{\frac{2}{2}}\right)$$

$$= \pm\left(\sqrt{9} - i\sqrt{1}\right)$$

$$= \pm(3-i)$$

(iii) $-15-36i$

Sol:-

Let $x+iy = -15-36i$

$\Rightarrow x = -15$, $y = -36 < 0$

$|-15-36i| = \sqrt{(-15)^2+(-36)^2}$

$= \sqrt{225+1296}$

$= \sqrt{1521}$

$= 39$

Applying Square root formula

$$\sqrt{-15-36i} = \pm\left(\sqrt{\frac{39-15}{2}} + \frac{(-36i)}{|36|}\sqrt{\frac{39+15}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{24}{2}} - i\sqrt{\frac{54}{2}}\right)$$

$$= \pm\left(\sqrt{12} - i\sqrt{27}\right)$$

$$= \pm\left(\sqrt{4\times3} - i\sqrt{9\times3}\right)$$

$$= \pm\left(2\sqrt{3} - i\,3\sqrt{3}\right)$$

(iv) $119 + 120i$

Sol:-

Let $x + iy = 119 + 120i$

$\Rightarrow x = 119 \quad y = 120 > 0$

$\sqrt{(119)^2 + (120i)^2} \Rightarrow \sqrt{14161 + 14400}$

$= \sqrt{28561}$

$= 169$

Applying Square root formula

$$\sqrt{119+120i} = \pm\left(\sqrt{\frac{169+119}{2}} + \frac{120i}{|120|}\sqrt{\frac{169-119}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{288}{2}} + i\sqrt{\frac{50}{2}}\right)$$

$$= \pm\left(\sqrt{144} + i\sqrt{25}\right)$$

$$= \pm\left(12 + 5i\right)$$

Q8 Find the square root of $13-20\sqrt{3}i$ and represent it on an Argand diagram.

Sol:-

Let $x+iy = 13-20\sqrt{3}i$

$\Rightarrow x = 13, \quad y = -20\sqrt{3} < 0$

$$= |13-20\sqrt{3}| = \sqrt{(13)^2+(20\sqrt{3})^2}$$

$$= \sqrt{169+400(3)}$$

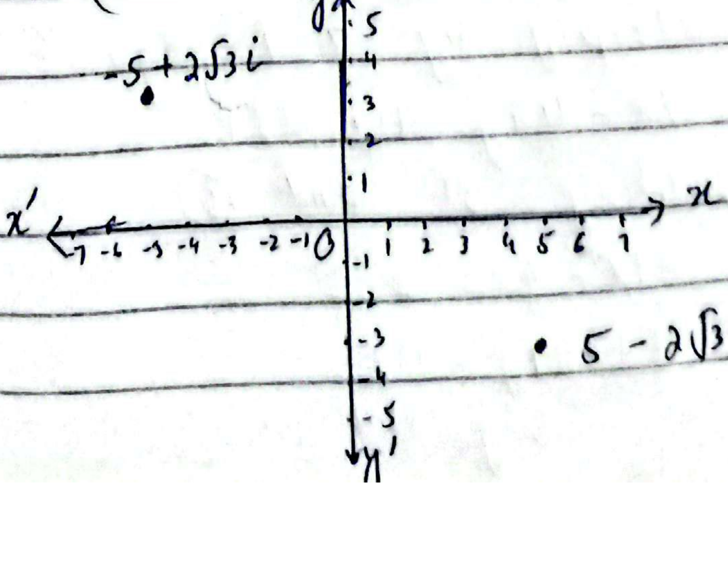
$$= \sqrt{169+1200} = \sqrt{1369} = 37$$

Applying the square root formula

$$\sqrt{13-20\sqrt{3}i} = \pm\left(\sqrt{\frac{37+13}{2}} + \frac{(-20\sqrt{3})i}{|20\sqrt{3}|}\sqrt{\frac{37-13}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{50}{2}} - i\sqrt{\frac{24}{2}}\right)$$

$$= \pm\left(\sqrt{25} - i\sqrt{12}\right) \Rightarrow \pm(5-2\sqrt{3}i)$$



Q9:- Find the real value of x and y if

$$(-7+i)(x+iy)+(-1-5i)=i(11-i)$$

Sol:-

$$(-7+i)(x+iy)+(-1-5i)=i(11-i)$$

$$-7(x+iy)+i(x+iy)+(-1-5i)=11i-i^2$$

$$-7x-7yi+xi+i^2y-1-5i=11i-(-1)$$

$$-7x-7yi+xi-y-1-5i=11i+1$$

$$(-7x-y-1)+(x-7y-5)i=1+11i$$

Equating real and img parts.

$$-7x-y-1=1 \rightarrow (1)$$

$$x-7y-5=11 \rightarrow (2)$$

from (1)

$$-7x-y=1+1$$

$$-7x-y=2 \rightarrow (3)$$

from (2)

$$x-7y=11+5$$

$$x-7y=16 \rightarrow (4)$$

Multiply by 7 to equ (4)

$$7x-49y=112 \rightarrow (5)$$

Adding (3) and (5)

$$\cancel{-7x}-y=2$$

$$\cancel{7x}-49y=112$$

$$-50y=114$$

$$y = \frac{114}{-50}$$

$$y = \frac{-57}{25}$$

Put $y = \frac{-57}{25}$ in eq4 (3)

$$-7x - \left(\frac{-57}{25}\right) = 2$$

$$-7x + \frac{57}{25} = 2$$

$$\frac{-175x + 57}{25} = 2$$

$$-175x + 57 = 50$$

$$-175x = 50 - 57$$

$$-175x = -7$$

$$x = \frac{7}{175}$$

$$\boxed{x = \frac{1}{25}}$$

Q10 Find the real value of x and y if $(5-2i)(x+iy)+3 = i(11-i)-4i$

Sol:-

$$(5-2i)(x+iy)+3 = i(11-i)-4i$$

$5(x+iy) - 2i(x+iy) + 3 = i(11-i) - 4i$

$5x + 5yi - 2xi - 2yi^2 + 3 = 11i - i^2 - 4i$

$5x + (5y-2x)i + 2y + 3 = 11i + 1 - 4i$

$(5x+2y+3) + (5y-2x)i = 1 + 7i$

Equating real and img parts.

$5x + 2y + 3 = 1$

$5x + 2y = 1 - 3$

$5x + 2y = -2 \longrightarrow (1)$

$5y - 2x = 7 \longrightarrow (2)$

Multiply by "2" and "5" to equ (1) and (2) respectively and Add them.

$10x + 4y = -4$

$-10x + 25y = 35$

$29y = 31$

$$\boxed{y = \frac{31}{29}}$$

Put $y = \frac{31}{29}$ in eq (1)

$5x + 2\left(\frac{31}{29}\right) = -2$

$5x + \frac{62}{29} = -2$

$$\frac{145x + 62}{29} = -2$$

$$145x + 62 = -58$$

$$145x = -58 - 62$$

$$145x = -120$$

$$x = \frac{-120}{145}$$

$$\boxed{x = \frac{-24}{29}} \quad (\div \text{ by } 5)$$

Q11:- Find the real value of u and v if $\frac{u-2}{2+i} + \frac{v-3}{2-i} = 4i$

Sol:-

$$\frac{u-2}{2+i} \times \frac{2-i}{2-i} + \frac{v-3}{2-i} \times \frac{2+i}{2+i} = 4i$$

$$\frac{(u-2)(2-i)}{(2+i)(2-i)} + \frac{(v-3)(2+i)}{(2-i)(2+i)} = 4i$$

$$\frac{u(2-i) - 2(2-i)}{(2)^2 - (i)^2} + \frac{v(2+i) - 3(2+i)}{(2)^2 - (i)^2} = 4i$$

$$\frac{2u - ui - 4 + 2i}{4 - (-1)} + \frac{2v + vi - 6 - 3i}{4 - (-1)} = 4i$$

$$\frac{(2u-4) + (2i - ui)}{4+1} + \frac{(2v-6) + (vi - 3i)}{4+1} = 4i$$

$$\frac{(2u-4)+(2-u)i}{5}+\frac{(2v-6)+(v-3)i}{5}=4i$$

$$\frac{(2u-4)+(2-u)i+(2v-6)+(v-3)i}{5}=4i$$

$$\frac{(2u-4+2v-6)+(2-u+v-3)i}{5}=4i$$

$$\frac{(2u+2v-10)+(-u+v-1)i}{5}=4i$$

$$\frac{2u+2v-10}{5}+\frac{(-u+v-1)i}{5}=4i$$

Equating real and img parts.

$$\frac{2u+2v-10}{5}=0$$

$$2u+2v-10=0$$

$$2u+2v=10$$

$$2(u+v)=10$$

$$u+v=\frac{10}{2}$$

$$u+v=5 \longrightarrow (1)$$

$$\frac{-u+v-1}{5}=4$$

$$-u+v-1=20$$

$-u+v = 20+1$

$-u+v = 21 \longrightarrow (2)$

Adding (1) and (2)

$$u+v=5$$
$$\underline{-u+v=21}$$
$$2v = 26$$
$$v = \frac{26}{2} \quad \boxed{v=13}$$

Put $v=13$ in (1)

$$u+13=5$$
$$u=5-13$$
$$\boxed{u=-8}$$

Q5 If $z_1 = 4+5i$ and $z_2 = a-2i$, find the real value of a such that $Re(z_1 z_2) = 20$

Sol:-

Given that

$$z_1 = 4+5i$$
$$z_2 = a-2i$$
$$z_1 . z_2 = (4+5i)(a-2i)$$
$$= 4(a-2i)+5i(a-2i)$$
$$= 4a - 8i + 5ai - 10i^2$$
$$= 4a + (5a-8)i + 10$$

$$z_1.z_2 = (4\alpha+10) + (5a-8)i$$

As

$$Re(z_1 z_2) = 20$$

So real part is $4a+10$

$$4\alpha+10 = 20$$

$$4\alpha = 20-10$$

$$4\alpha = 10$$

$$\alpha = \frac{10}{4}$$

$$\boxed{\alpha = \frac{5}{2}}$$