Function And Graph

Ex # 2.1

Q1_ Given that:

a): $f(x) = x^2 - 1$

b): $f(x) = \sqrt{2x+3}$

Find:

i. $f(-3)$ ii. $f(0)$

iii. $f(x-2)$ iv. $f(x^2+3)$?

Sol_ (a): $f(x) = x^2 - 1 \longrightarrow (i)$

i_ As

$$f(x) = x^2 - 1$$

Now put $x = -3$

$$f(-3) = (-3)^2 - 1$$

$$= 9 - 1 \qquad (\because (-3)^2 = 9)$$

$$= 8$$

ii_ As, $f(x) = x^2 - 1$

Now put $x = 0$

$$f(0) = (0)^2 - 1$$

$$= 0 - 1$$

$$= -1$$

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iii_ As

$$f(x) = x^2 - 1$$

Now replacing x by $x-2$,

$$f(x-2) = (x-2)^2 - 1$$
$$= \left((x)^2 - 2(x)(2) + (2)^2\right) - 1$$
$$= x^2 - 4x + 4 - 1$$
$$= x^2 - 4x + 3$$

iv_ As

$$f(x) = x^2 - 1$$

Now replacing x by x^2+3,

$$f(x^2+3) = (x^2+3)^2 - 1$$
$$= \left((x^2)^2 + 2(x^2)(3) + (3)^2\right) - 1$$
$$= x^4 + 6x^2 + 9 - 1$$
$$= x^4 + 6x^2 + 8$$

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(b): $f(x) = \sqrt{2x+3}$

i. As

$$f(x) = \sqrt{2x+3}$$

Now put $x = -3$

$$f(-3) = \sqrt{2(-3)+3}$$
$$= \sqrt{-6+3}$$
$$= \sqrt{-3}$$
$$= \sqrt{3}\, i$$

ii. As

$$f(x) = \sqrt{2x+3}$$

Now put $x=0$

$$f(0) = \sqrt{2(0)+3}$$
$$= \sqrt{0+3}$$
$$= \sqrt{3}$$

iii. As

$$f(x) = \sqrt{2x+3}$$

Now replace x by $x-2$

$$f(x-2) = \sqrt{2(x-2)+3}$$
$$= \sqrt{2x-4+3}$$
$$= \sqrt{2x-1}$$

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iii) As

$$f(x) = \sqrt{2x+3}$$

Now replace x by x^2+3,

$$f(x^2+3) = \sqrt{2(x^2+3)+3}$$

$$= \sqrt{2x^2+6+3}$$

$$= \sqrt{2x^2+9}$$

$\longleftrightarrow$

Q2. Find: $\dfrac{f(a+h)-f(a)}{h}$ and simplify where,

i) $f(x) = 4x+7$?

As,

$$f(x) = 4x+7 \longrightarrow (i)$$

Now put $x = a$ in eq(i)

$$f(a) = 4a+7 \longrightarrow (A)$$

Now put $x = a+h$ in eq(i)

$$f(a+h) = 4(a+h)+7$$

$$= 4a+4h+7 \longrightarrow (B)$$

Now by using eq(A) and eq(B),

$$\frac{f(a+h)-f(a)}{h} = \frac{(4a+4h+7)-(4a+7)}{h}$$

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$$= \frac{4a + 4h + 7 - 4a - 7}{h}$$

$$= \frac{4h}{h}$$

$$= 4$$

11. $f(x) = \sin x$?

As.

$$f(x) = \sin x \longrightarrow (ii)$$

Now putting $x = a$ in eq(ii),

$$f(a) = \sin a \longrightarrow (A)$$

Now putting $x = a+h$ in eq(ii),

$$f(a+h) = \sin(a+h) \longrightarrow (B)$$

Now using eq(A) and eq(B),

$$\frac{f(a+h) - f(a)}{h} = \frac{\sin(\overset{\alpha}{a+h}) - \sin \overset{\beta}{a}}{h}$$

$$\left(\because \sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \right)$$

$$= \frac{1}{h}\left(2\cos\left(\frac{a+h+a}{2}\right) \sin\left(\frac{a+h-a}{2}\right) \right)$$

$$= \frac{2}{h} \cos\left(\frac{2a+h}{2}\right) . \sin\left(\frac{h}{2}\right)$$

$$= \frac{2}{h} . \cos\left(a + \frac{h}{2}\right) . \sin\left(\frac{h}{2}\right)$$

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iii. $f(x) = x^3 + x^2 - 1$?

As

$$f(x) = x^3 + x^2 - 1 \quad \text{—— (iii)}$$

Now put $x = a$ in eq(iii)

$$f(a) = a^3 + a^2 - 1 \quad \text{—— (A)}$$

Now put $x = a + h$ in eq(iii)

$$f(a+h) = (a+h)^3 + (a+h)^2 - 1$$

Now applying formula of cube & square.

$$= (a^3 + h^3 + 3a^2h + 3ah^2) + (a^2 + h^2 + 2ah) - 1$$

$$f(a+h) = a^3 + h^3 + 3a^2h + 3ah^2 + a^2 + h^2 + 2ah - 1 \longrightarrow (B)$$

Now using eq(A) and eq(B) ;

$$\frac{f(a+h) - f(a)}{h} =$$

$$\frac{(a^3 + h^3 + 3ah^2 + 3a^2h + a^2 + h^2 + 2ah - 1) - (a^3 + a^2 - 1)}{h}$$

$$= \frac{\cancel{a^3} + h^3 + 3ah^2 + 3a^2h + \cancel{a^2} + h^2 + 2ah - \cancel{1} - \cancel{a^3} - \cancel{a^2} + \cancel{1}}{h}$$

$$= \frac{h^3 + 3ah^2 + 3a^2h + h^2 + 2ah}{h}$$

$$= \frac{\cancel{h}(h^2 + 3ah + 3a^2 + h + 2a)}{\cancel{h}}$$

$$= h^2 + 3ah + h + 3a^2 + 2a \quad \text{Ans.}$$

∵ Formula:

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Date:

iv. $f(x) = \tan x$?

As

$$f(x) = \tan x \longrightarrow (iv)$$

Now put $x = a$ in eq(iv)

$$f(a) = \tan(a) \longrightarrow (A)$$

$$= \frac{\sin a}{\cos a}$$

Now put $x = a+h$ in eq(iv)

$$f(a+h) = \tan(a+h)$$

$$= \frac{\sin(a+h)}{\cos(a+h)} \longrightarrow (B)$$

Now using eq(A) and eq(B)

$$\frac{f(a+h) - f(a)}{h} = \frac{1}{h}\left(f(a+h) - f(a)\right)$$

$$= \frac{1}{h}\left(\frac{\sin(a+h)}{\cos(a+h)} - \frac{\sin a}{\cos a}\right)$$

Taking L.C.M

$$= \frac{1}{h}\left(\frac{\sin(a+h)\cos a - \cos(a+h)\sin a}{\cos(a+h)\cdot\cos a}\right)$$

$$= \frac{1}{h}\left(\frac{\sin((a+h)-a)}{\cos(a+h)\cdot\cos a}\right)$$

$$= \frac{1}{h}\,\frac{\sin(\cancel{a}+h-\cancel{a})}{\cos(a+h)\cdot\cos a}$$

$$= \frac{1}{h}\cdot\frac{\sin h}{\cos(a+h)\cdot\cos a}$$

FORMULA: $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Date:

Q3_ Express the following:

a): The area "A" of square as a function of its perimeter "P" ?

Sol_ Let x be the side of square.

As we know that

Area = lenght × Width

$A = x \cdot x$

$\boxed{A = x^2} \longrightarrow (I)$

∵ In square
$l = \omega \; (= x)$

As

Perimeter = Sum of all sides

$P = x + x + x + x$

$P = 4x$

$\frac{P}{4} = x \quad \Rightarrow \quad \boxed{x = P/4}$

Now putting value of x in eq(II)

$\Rightarrow \quad A = \left(\frac{P}{4}\right)^2$

$A = P^2/16$ Ans.

Date:

b): The Circumference 'C' of a circle as a function of its area 'A'?

Sol- As we know that.

circumference ⇒ $C = 2\pi r \longrightarrow (A)$

And

area ⇒ $A = \pi r^2$

(OR) $\pi r^2 = A$

$r^2 = \frac{A}{\pi}$

Taking square root on b/s

$\sqrt{r^2} = \sqrt{\frac{A}{\pi}}$

$$\boxed{r = \sqrt{\frac{A}{\pi}}}$$

Now putting value of r in eq(A)

$C = 2\pi\sqrt{\frac{A}{\pi}}$

$\because \pi = \sqrt{\pi^2}$

$= \sqrt{\pi . \pi}$

$= \sqrt{\pi}.\sqrt{\pi}$

$= 2\sqrt{\pi}\cancel{\sqrt{\pi}}\frac{\sqrt{A}}{\cancel{\sqrt{\pi}}}$

$= 2\sqrt{\pi}\sqrt{A}$

$= 2\sqrt{\pi A}$

Ans.

Date:

c): The surface area 'S' of a cube as a function of volume 'V':

Sol- As we know that

$A = 6 \cdot l \cdot w$ ($\because$ Cube has 6 sides)

Let x be the side of cube. So,

$A = 6x \cdot x$

$A = 6x^2 \longrightarrow (I)$

And volume (Formula) is: (for cube)

$V = l \cdot w \cdot h$

$V = x \cdot x \cdot x$

$V = x^3$

(OR) $x^3 = V$

Taking cube root

$(x^3)^{1/3} = (V)^{1/3}$

$\boxed{x = V^{1/3}}$

Now putting value of x in eq(I)

$A = 6(V^{1/3})^2$

$A = 6V^{2/3}$

Ans.

Graphs are not mendatory.

Date:

Q4_ Find the domain and range of the function 'g' define below:

i). $g(x) = 5 - x$

Sol_ at $x = 0 \Rightarrow g(0) = 5 - 0 = 5$

at $x = 1 \Rightarrow g(1) = 5 - 1 = 4$

at $x = 2 \Rightarrow g(2) = 5 - 2 = 3$

at $x = 3 \Rightarrow g(3) = 5 - 3 = 2$

at $x = 4 \Rightarrow g(4) = 5 - 4 = 1$

at $x = 5 \Rightarrow g(5) = 5 - 5 = 0$

at $x = 6 \Rightarrow g(6) = 5 - 6 = -1$

at $x = 7 \Rightarrow g(7) = 5 - 7 = -2$

And

at $x = -1 \Rightarrow g(-1) = 5 - (-1) = 6$

at $x = -2 \Rightarrow g(-2) = 5 - (-2) = 7$

$\vdots \qquad \vdots$

So,

Domain $= \mathbb{R} = (-\infty, \infty)$

Range $= \mathbb{R} = (-\infty, \infty)$

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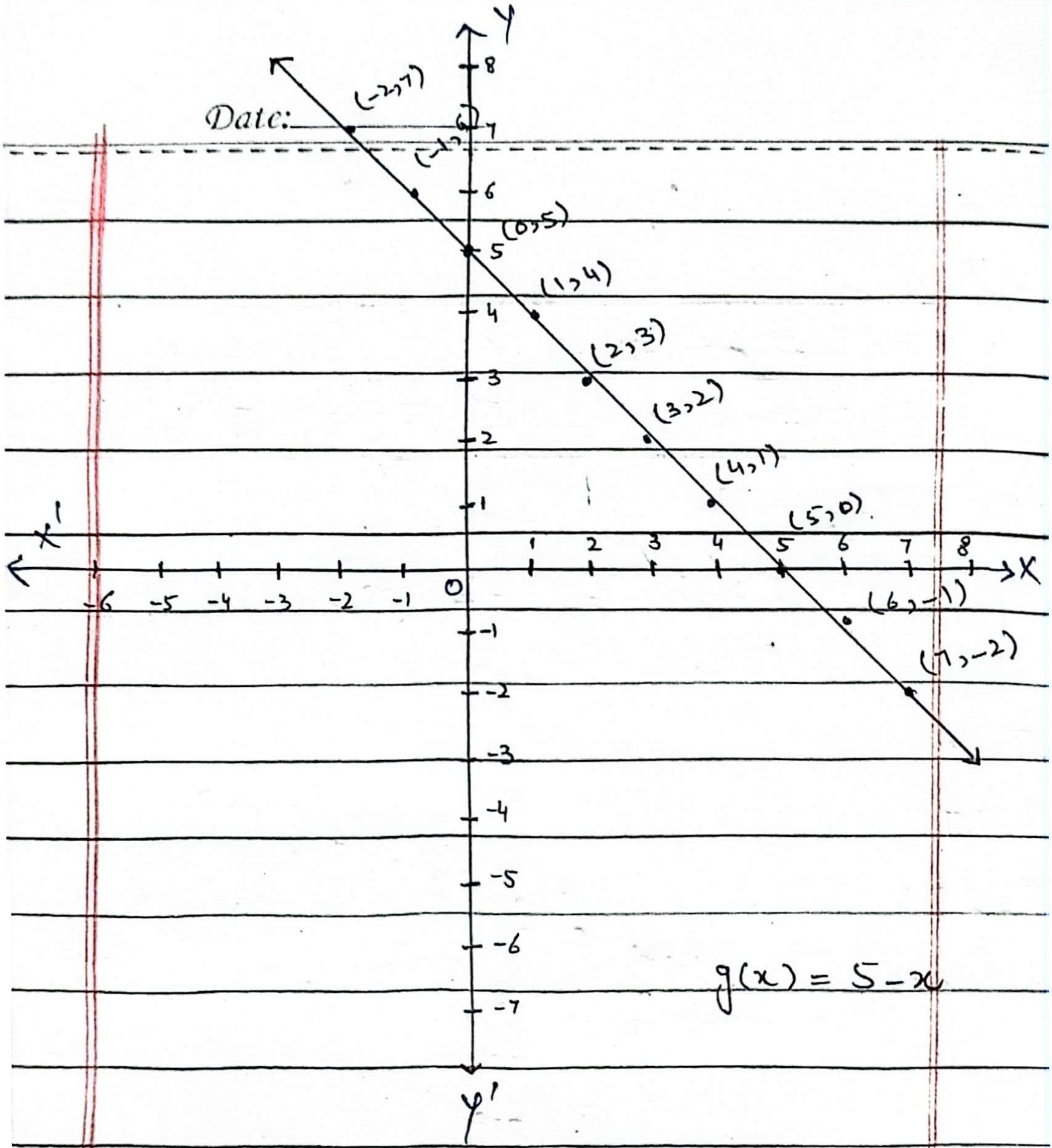

Y
X'
X
Y'
O
(-2,7)
(-1,6)
(0,5)
(1,4)
(2,3)
(3,2)
(4,1)
(5,0)
(6,-1)
(7,-2)
g(x) = 5-x

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ii. $g(x) = \sqrt{x+2}$

Sol.

اس کے بعد Imag جواب آرہے

ایسا Domain میں نہیں شامل ⟵

x	$\dots -4$	-3	-2	-1	0	1	2	3	$\dots$
$g(x) = \sqrt{x+2}$	$\dots \sqrt{2}\,i$	i	0	$\sqrt{1} = 1$	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4} = 2$	$\sqrt{5}$	$\dots$

Range ⟹ zero se aage (+)ve answer hm

So,

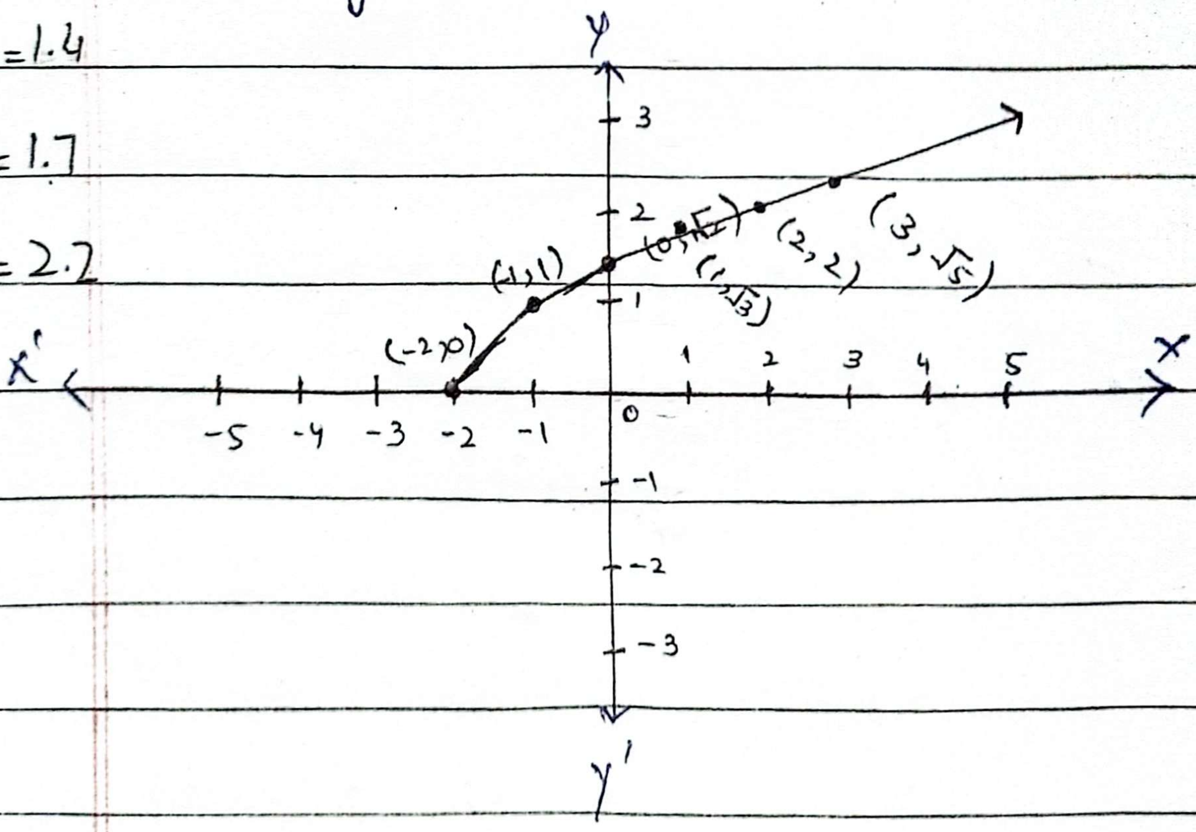
$$\text{Domain} = [-2, \infty)$$

$$\text{Range} = [0, \infty)$$

$\sqrt{2} = 1.4$

$\sqrt{3} = 1.7$

$\sqrt{5} = 2.2$

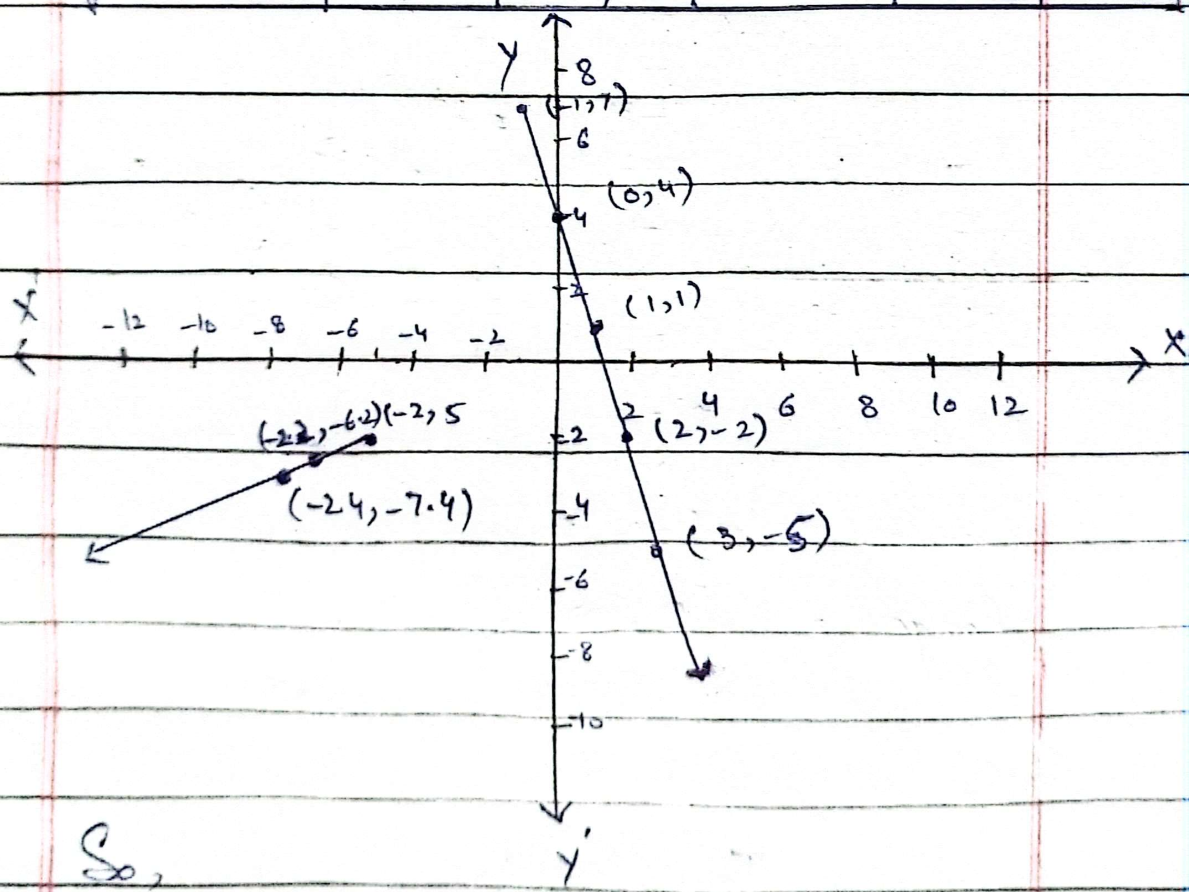


Date:

iii- $g(x) = \begin{cases} 6x+7 & , x \le -2 \\ 4-3x & , x > -2 \end{cases}$?

$x \le -2$	-2	-2.2	-2.4	-2.6
$g(x) = 6x+7$	-5	-6.2	-7.4	-8.6

$x > -2$	-1	0	1	2	3
$g(x) = 4-3x$	7	4	1	-2	-5



So,

Domain = $(-\infty, +\infty)$

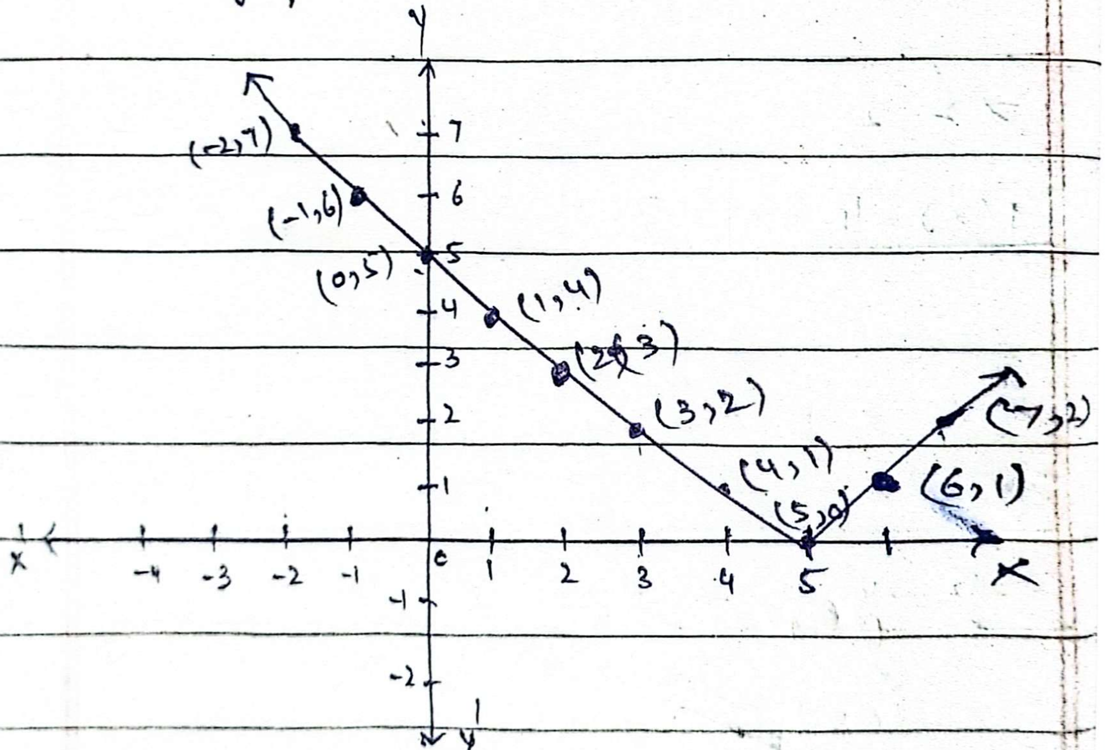
Range = $(-\infty, 7]$

iv. $g(x) = |x-5|$

Sol

x	-2	-1	0	1	2	3
$g(x)$	7	6	5	4	3	2

So graph is :



Domain = $(-\infty, +\infty)$

Range = $[0, +\infty)$

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v. $g(x) = \dfrac{x+2}{3-x}$

$g(x)$ is undefined at $x = 3$.

And $y = \dfrac{x+2}{3-x}$ $(\because g(x) = y)$

$$(3-x)y = x+2$$
$$3y - xy = x+2$$
$$3y - 2 = x + xy$$

(OR)

$$x(1+y) = 3y-2$$
$$x = \frac{3y-2}{1+y}$$

And x is un-defined for $y = -1$.

So,

$$\text{Domain} = \mathbb{R} - \{3\}$$
$$\text{Range} = \mathbb{R} - \{-1\}$$

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Q5_ Given $f(x) = x^3 - ax^2 + bx + 1$.
If $f(2) = -3$ & $f(-1) = 0$.
Find a & b ?

Sol_ Given

$$f(x) = x^3 - ax^2 + bx + 1 \longrightarrow \text{(I)}$$

And

$$f(2) = -3 \quad \text{— (A)}$$

$$f(-1) = 0 \quad \text{— (B)}$$

Now put $x = 2$ in eq (I)

$$f(2) = (2)^3 - a(2)^2 + b(2) + 1$$

$$= 8 - 4a + 2b + 1$$

$$= -4a + 2b + 9 \longrightarrow \text{(C)}$$

Now put $x = -1$ in eq (I).

$$f(-1) = (-1)^3 - a(-1)^2 + b(-1) + 1$$

$$= -\cancel{1} - a - b + \cancel{1}$$

$$= -a - b \longrightarrow \text{(D)}$$

By Compairing eq (A) & eq (C),

$$-4a + 2b + 9 = -3$$

$$-4a + 2b + 9 + 3 = 0$$

$$\boxed{-4a + 2b + 12 = 0} \longrightarrow (1)$$

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Now by compairing eq(B) & eq(D)

$$\boxed{-a - b = 0} \longleftarrow (2)$$

Now we solve equations to find a & b.

From eq(2)

$$-a - b = 0$$

$$\boxed{a = -b} \longleftarrow (3)$$

put in eq(1)

$$-4(-b) + 2b + 12 = 0$$

$$4b + 2b + 12 = 0$$

$$6b + 12 = 0$$

$$6b = -12$$

$$b = -12/6$$

$$\boxed{b = -2}$$

Now putting value of 'b' in eq(3)

$a = -(-2)$ $\boxed{a = +2}$

So,

$a = +2$

$b = -2$ Ans

Date:

Q6. A stone falls from height of 60m on ground. the height h after x sec. is given

$$h(x) = 40 - 10x^2$$

i/ what is the height when

a) $x = 1$ sec b) $x = 1.5$ s

c) $x = 1.7$ s

ii/ When does stone strike the ground?

Sol. i/ Given

$$h(x) = 40 - 10x^2$$

a) at $x = 1$s

$$h(1) = 40 - 10(1)^2$$

$$= 40 - 10$$

$$h(1) = 30\,m$$

b) at $x = 1.5$ s

$$h(1.5) = 40 - 10(1.5)^2$$

$$= 40 - 10(2.25)$$

$$= 40 - 22.5$$

$$= 17.5$$

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c) at $x = 1.75$

$$h(1.7) = 40 - 10(1.7)^2$$
$$= 40 - 10(2.89)$$
$$= 40 - 28.9$$
$$= 11.1$$

ii/ $h(x) =$ height after x seconds.

When stone strike the ground.

$$h(x) = 0$$

Therefore

$$40 - 10x^2 = 0$$
$$\cancel{-}10x^2 = \cancel{-}40$$
$$x^2 = \frac{4\cancel{0}}{1\cancel{0}}$$
$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = 2$$

$\left[\begin{array}{l} \because \sqrt{4} = \pm 2 \\ \text{As time can't be negtive So neglect} \\ -2 \end{array} \right]$

Date: ____________

Q7_ Consider the function

$$f(x) = 3x - 5$$

i_ Determine Domain & Range.

ii_ Is function f one-one. Justify.

iii_ Is function onto if codomain is real numbers. Explain?

Sol_ i)_ As

$$f(x) = 3x - 5$$

x	-2	-1	0	+1	+2	3
$f(x)$	-11	-8	-5	-2	1	4

So,

Domain $= (-\infty, +\infty)$

Range $= (-\infty, +\infty)$

ii)_ A function is one-one if,

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad x_1 = x_2$$

As $f(x) = 3x - 5$

So $f(x_1) = 3x_1 - 5$

$f(x_2) = 3x_2 - 5$

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Suppose

$$f(x_1) = f(x_2)$$

$$3x_1 - \cancel{5} = 3x_2 - \cancel{5}$$

$$3x_1 = 3x_2$$

Dividing b/s by 3,

$$\frac{3x_1}{3} = \frac{3x_2}{3}.$$

$$x_1 = x_2$$

This implies that function is one-one because equal outputs implies equal inputs.

ii) A function is said to be onto if every element in codomain Y there exist atleast one pre-image in Domain X.

As $f(x) = 3x - 5$

$$y = 3x - 5 \quad \because f(x) = y$$

$$y + 5 = 3x$$

$$\Rightarrow x = \frac{y+5}{3}, \text{ where codomain is real numbers.}$$

So, for each y there exist $\frac{y+5}{3}$ is R such that

$$f\left(\frac{y+5}{3}\right) = y.$$ Hence f is onto.

Date:

Q8 Let $f: R \to R$ be defined by: $f(x) = \frac{2x-3}{x+1}$

i. Find domain & range.
ii. Determine whether f is onto.
iii. Prove f is one-one.

Sol i/ As

$$f(x) = \frac{2x-3}{x+1}$$

As function is un-defied at -1. So,

Domain = $R - \{-1\}$
Range = $R - \{2\}$

ii/ A function is onto if for every value of codomain (y) there exist a preimage (x).

Let $f(x) = \frac{2x+3}{x+1} = y$

$$2x+3 = y(x+1)$$
$$2x+3 = yx+y$$
$$2x - yx = y-3$$

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$$x(2-y) = y-3$$

$$\Rightarrow \quad x = \frac{y-3}{y-2}$$

x is un-defined at $y=2$.
As for $y=2$ there doesn't exist any x. So $f(x)$ is not onto.

iii/ To prove if

$$f(x_1) = f(x_2)$$

then $x_1 = x_2$.

Let

$$f(x_1) = f(x_2)$$

$$\frac{2x_1-3}{x_1+1} = \frac{2x_2-3}{x_2+1}$$

$$(2x_1-3)(x_2+1) = (2x_2-3)(x_1+1)$$

$$\cancel{2x_1x_2} - 3x_2 + 2x_1 - \cancel{3} = \cancel{2x_1x_2} - 3x_1 + 2x_2 - \cancel{3}$$

$$-3x_2 + 2x_1 = -3x_1 + 2x_2$$

$$2x_1 + 3x_1 = 2x_2 + 3x_2$$

$$5x_1 = 5x_2$$

$$\Rightarrow \quad x_1 = x_2$$

Date:

Q9 Consider $f: R^+ \to R^+$, defined by: $f(x) = e^{-x}$. Show f is bijective?

Sol We have to prove that $f(x)$ is one-to-one and onto.

Let

$$f(x_1) = f(x_2)$$

$$e^{-x_1} = e^{-x_2}$$

Apply 'ln' on b/s,

$$\ln e^{-x_1} = \ln e^{-x_2}$$

$$-x_1 \ln e = -x_2 \ln e$$

$$\cancel{-}x_1 = \cancel{-}x_2$$

$$x_1 = x_2$$

$\Rightarrow$ f is one-to-one.

Now let,

$$f(x) = e^{-x} = y$$

$$-x = \ln y$$

$$x = -\ln y$$

So every value of y there exist x.

Since f is one-one & onto.

So f is bijective

Date:

Q10- Let $g: R \to R$ be given by $g(x) = x^3 - 3x$.
Determine if $g(x)$ is injective or surjective?

Sol- To prove that $g(x)$ is injective we have to prove that if

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad x_1 = x_2$$

let

$$g(x_1) = g(x_2)$$

$$x_1^3 - 3x_1 = x_2^3 - 3x_2$$

$$x_1(x_1^2 - 3) = x_2(x_2^2 - 3)$$

$$\Rightarrow \quad x_1 \neq x_2$$

So $g(x)$ is not injective.

Now let

$$g(x) = x^3 - 3x = y$$

$$x^3 - 3x - y = 0$$

This equation have cube root so for every y there exist atleast one pre-image $x \in R$.

Hence $g(x)$ is Surjective.